Documentation of FullSWOF_1D-transfer v2.00.01 (2023-03-13)

The FullSWOF development team

2021-05-02

Contents

1 Presentation of the Transfer equations

The Transfer equations model the evolution of several (here N) classes of particles, see Nouhou Bako et al. (2021). In FullSWOF_1D-transfer, the Shallow Water equations are coupled with the following Transfer equations:

$$\begin{cases} \dfrac{\partial hc_i}{\partial t} + \dfrac{\partial (qc_i)}{\partial x} = \dfrac{1}{ts_i}(g(M_i) - hc_i) + S_1 \quad i = 1, 2, \ldots, N \\ \alpha \dfrac{\partial M_i}{\partial t} = -\dfrac{1}{ts_i}(g(M_i) - hc_i) + S_2 \quad i = 1, 2, \ldots, N \end{cases} \tag{1}$$

where, for each class i of particles, c_i [kg m^{-2}] and M_i [kg m^{-1}] are the particle concentration in the flow and in the exchange layer respectively, h [m] is the water layer depth and q [m^2 s^{-1}] is the water flux per unit width. The parameters $g(M_i)$ [kg m^{-1}] and ts_i [s] are respectively the equilibrium function and the relaxation time. The terms S_1 (see sections 2.7-2.9) and S_2 (see section 2.10) [kg s^{-1} m^{-1}] are the supplies for the flow concentration and the exchange layer respectively and α is a constant coefficient.

In the following, the configuration of parameters specific to chemical transport or to sedimentation will be emphasized.

Remark 1 *At this stage of development,*

- *only the HLL flux and, at the second order, MUSCL reconstruction with the slope limiter minmod are used. A generalization to let the user choose the flux and / or the reconstruction and / or the slope limiter is still in progress,*
- *the topography is not modified by the Transfer equations.*

About the regular compilation and benchmarking, please refer to the documentation of FullSWOF_1D (Documentation.pdf in the doc directory).

To run the resolution of the Transfer equations without the Shallow-Water equations, you must set the withFS1Dwater tag to no in the Sources/libtransfer/make_config_transfer file and run the code from the libtransfer directory using:

```
cd Sources/libtransfer/
make clean
make
```

In this case, the values of the water height and water velocity must be specified at the end of the transfer parameter file, as well as the size of the domain, see section 3.

2 Inputs parameters

Remark 2 *For the moment, the particles, as well as the flow, go from the left part to the right of the domain. A generalization of these conditions will be performed in the forthcoming versions.*

You can set the values of most of parameters in the transfer.txt file, located in the Inputs directory. If a parameter is not set, we either affect a default value or stop the program (depending on the type of parameter), and the code will return either a warning or an error message.

2.1 General parameters

The Transfer equations (1) depend on N, the number of classes of particles. This value must be specified with the `<Nc>` tag.

The duration of the simulation is fixed by the flow parameters (see also section 3), but you can choose the number of times where the concentrations are saved in `<nbtimes_conc>`.

You can choose between several numerical schemes: the order in time is given by the flow parameters, or, if the Transfer equations are not coupled with FullSWOF_1D, a first order in time scheme is performed in the file FullSWOF_transfer_1D.cpp. But you can choose the spatial order to solve the Transfer equations (1) with `<order_rec>`:

- case 1 to have a first order in space scheme,
- case 2 to have a second order in space scheme, with MUSCL reconstruction of variables and minmod slope limiter.

You can also choose to save the concentration at each time step, at one space point. More precisely, you can set the value of `<Temp>`:

- case 0 to disable the creation of the file,
- case 1 to have a record at each time step at the point `<X_temp>`,

see section 4 for details.

2.2 Inflow concentrations

For the left boundary (*i.e* $x = 0$) you may have to specify the inflow concentration `<cin>` for each class i as follows:

- case 0 to have `<cin>=0` for all the classes.
- case 1 to impose the same `<cin>` for all the classes; in this case `<cincoef>` is the value of the inflow concentration.
- case 2 to load the inflow concentration from a file. The file must be in ASCII and must contain two columns:
  - the first one is the name of the class.
  - the second one is the value of the inflow concentration for the corresponding class.

  The **name of your inflow concentration file** `<cin_NF>` should be indicated and this file must be in the Inputs directory.

You should also set the starting time `<Tin>` and the ending time `<Tstop>` to indicate when the inflow concentration begins and when it stops ($T_{in} < t < T_{stop}$).

2.3 Initial conditions

The kind of initialization for the concentrations c_i and M_i is set with `<cM_init>`:

- case 0 to impose $c_i = 0$ [kg m^{-2}] and $M_i = 0$ [kg m^{-1}] for all the classes.
- case 1 to have the same initial concentrations for all the classes; for this choice specify the values of `<cicoef>` and `<Micoef>` for respectively the flow concentration c_i and the exchange layer M_i.
- case 2 to load the initial concentrations from an ASCII file. According to the number of class, the file must follow the format "$x\ c_1\ M_1\ c_2\ M_2\ \ldots c_i\ M_i\ \ldots c_N\ M_N$".
  - the first column must contain the x values following the FullSWOF_1D format.
  - the even columns must contain the c_i values with respect to the class i.
  - the next odd columns must contain the M_i values with respect to the class i.

  You should specify the **name of the file** `<cM_NF>` and put it in the Inputs directory.

  Remark 3 *If the values of the c_i and/or M_i are different depending on i but does not depend on x (the values are homogeneous in space) you can generate your file thanks to the* GenerateConstantFile.sh *script (in the* bin *folder).*

2.4 Coefficient of the concentration M_i

The coefficient α of the concentration M_i in Equation (1) should be specified with `<Alphacoef>`.

2.5 Relaxation parameters

The relaxation time $ts_i(s)$ is calculated by using a relaxation parameter rp_i. The relaxation parameter rp_i should be chosen through `<Rpi>`:

- case 1 to set the same `<Rpi>` for all the classes; in this case the value of `<Rpicoef>` must be specified.
- case 2 to load the relaxation parameter from a file. The file must follow the same format as in section 2.2, case 2. The **name of your file** `<Rpi_NF>` should be indicated and this file must be in the Inputs directory.

The relaxation time is chosen with `<Tsi>` (take care to take a small time step if ts_i is small):

- case 0 to have $ts_i = rp_i$.
- case 1 to have $ts_i = h \cdot rp_i$, where h is the water layer depth.
- case 2 to choose a time used for chemical transfer $ts_i = h/(rp_i(R - R_{0chem}) + I)$, where I [m s^{-1}], R [m s^{-1}] and R_{0chem} [m s^{-1}] are respectively the infiltration, the rain intensity indicated in the file parameters.txt and the rain intensity threshold for chemical transfer. For this choice, you must indicate the value of the rain intensity threshold for chemical transfer `<R0_chem>`.

2.6 Equilibrium function

The equilibrium function $g(M_i)$ in Equation (1) can be chosen between the following functions:

$$g(M_i) = K_{ex} M_i \tag{2a}$$

$$g_+(M_i) = \frac{K_{ex} M_i}{C_a + \sum_i K_{ex} M_i} \tag{2b}$$

$$g_-(M_i) = \frac{K_{ex} M_i}{C_a - \sum_i K_{ex} M_i} \tag{2c}$$

where the coefficient K_{ex} is the exchange type of the equilibrium function (see section 2.6.2) and C_a is a characteristic concentration (see section 2.6.1). The kind of equilibrium function is set with `<GMi>`:

- case 1 to choose the linear function $g(M_i)$.
- case 2 to choose the nonlinear function $g_+(M_i)$.
- case 3 to choose the nonlinear function $g_-(M_i)$.

2.6.1 Characteristic concentration

For the choice of $g_+(M_i)$ and $g_-(M_i)$, the value of the characteristic concentration `<CAcoef>` must be indicated.

2.6.2 Exchange type

The exchange type of the equilibrium function K_{ex} is calculated using the parameter of the equilibrium function K_i as follows:

$$K_{ex} = K_i \tag{3a}$$

$$K_{ex} = K_i \cdot ts_i \cdot (R - R_0)_+ \tag{3b}$$

$$K_{ex} = K_i \cdot (q - q_0)_+ \tag{3c}$$

$$K_{ex} = K_i \cdot ts_i \cdot (R - R_0)_+ + K_i^+ \cdot (q - q_0)_+ \tag{3d}$$

$$K_{ex} = K_i \cdot ts_i \cdot (R - R_{0chem})_+ \tag{3e}$$

where the + subscript stands for the positive part, *i.e.* the maximum between the quantity and zero. The choice of the exchange type K_{ex} is set with the value of `<Kex_type>`:

- case 0 to choose Equation (3a).
- case 1 to choose Equation (3b). Choose this equation for rain exchange and indicate the value of the rain intensity threshold `<R0>`.
- case 2 to choose Equation (3c), where q_0 [$\mathrm{m^2\,s^{-1}}$] is the water flux threshold per unit width that must be indicated by setting the value of `<q0>`. This choice is appropriated for the flow exchange.
- case 3 to choose Equation (3d) when rain and flow exchange are both involved. The values of the coefficients of `<R0>` and `<q0>` must be indicated. You should also indicate the value of K_i^+ with the tag `<Ki2>`, see section 2.6.3.
- case 4 to choose Equation (3e) when chemical transfer is concerned. For this choice, you must have previously indicated the rain intensity threshold for chemical transfer `<R0_chem>`.

Note that if you set `<Tsi>`=1 (which means $tsi = h \cdot rp_i$), you cannot choose `<Kex_type>`=4. Likewise the choice of `<Tsi>`=2 is not consistent with `<Kex_type>`=1, 2 or 3. This avoids any conflict between chemical and sediment parameters.

2.6.3 Equilibrium parameter

The parameters K_i and K_i^+ of the equilibrium function (3a)–(3e) are set with `<Ki>` and `<Ki2>` respectively:

- case 1 to choose the same parameters for all the classes; in this case the values of `<Kicoef>` and eventually (for Equation (3d)) `<Kicoef2>` must be specified.

- case 2 to load the parameters of the equilibrium function from files which have the same format as the file in section 2.2, case 2. The **names of the parameter files** `<Ki_NF>` (and `<Ki_NF2>`) should be indicated and put in the Inputs directory.

2.7 Distributed supply for the flow concentration

A supply can be added in the flow concentration in each x-cell with one of the following equations:

$$D_s = Ds_{coef}, \quad \text{for} \quad TD_{start} < t < TD_{end}, \tag{4a}$$
$$D_s = Ds_i(x), \quad \text{for} \quad TD_{start} < t < TD_{end}, \tag{4b}$$

where Ds_{coef} [$\mathrm{kg\,s^{-1}\,m^{-1}}$] is a constant coefficient, TD_{start} and TD_{end} are respectively starting and ending times of the supply.

The choice of D_s is done with `<Ds>`:

- case 0 to set the supply to zero `<Ds>`=0.
- case 1 to choose Equation (4a) which add the same supply for all the classes; for this choice specify the value of the coefficient `<Dscoef>`.
- case 2 to choose Equation (4b) where the value of $Ds_i(x)$ [$\mathrm{kg\,s^{-1}\,m^{-1}}$] are contained in an ASCII file. According to the number of class, the file must follow the format "$x\ Ds_1\ Ds_2 \ldots Ds_i \ldots Ds_N$":
  - the first column must contain the x values following the FullSWOF_1D format.
  - the other columns must contain the Ds_i values with respect to the class i.

  You should specify the **name of the supply file** `<Ds_NF>` ant put it in the Inputs directory.

  Remark 4 *If the values of the Ds_i are different depending on i but does not depend on x (i.e. the values are homogeneous in space) you can generate your file thanks the* GenerateConstantFile.sh *script (in the* bin *folder).*

For the cases 1 and 2, you must indicate the starting time `<TD_start>` and ending time `<TD_end>` of the supply.

2.8 Rain supply for the flow concentration

The rain supply is calculated using the equation:

$$R_{det} = P_i A_d (R - R_0)^B \left(1 - \left(\frac{\sum_i M_i}{M_{dt^*}}\right)\right), \quad \text{for} \quad t < TR_{end} \tag{5}$$

where P_i is the proportion of each class i in the initial soil, A_d is the detachability of the initial soil, B is the exponent of the rain excess, M_{dt^*} the mass for complete shielding and TR_{end} the depletion time of the rain supply. The exponent B can be chosen between the following equations:

$$B = B_{coef} \tag{6a}$$
$$B = 0.8 \exp(0.06h) \tag{6b}$$

where B_{coef} is a constant exponent, usually equal to 1.

The rain supply can be set using `<Rdet>` as:

- case 0 to set the rain supply to zero `<Rdet>`=0.
- case 1 to set the rain supply with Equation (5). In this case, the rain intensity threshold R_0 [$\mathrm{m\,s^{-1}}$]and the coefficient A_d must be indicated using `<R0>` and `<Adcoef>`. You can choose the exponent B with `<B>` as:
  - case 0 to choose Equation (6a). For this choice, you must indicate the value of the exponent `<Bcoef>`.
  - case 1 to choose Equation (6b).

  The depletion time TR_{end} and the mass for complete shielding M_{dt^*} must also be indicated in `<TR_end>` and `<Mscoef>` respectively. You should specify the proportion of each class `<Pi>` as follows:

- case 1 to have the same proportion for all the classes; in this case the value of `<Picoef>` must be specified.
- case 2 to load the proportion for each class from a file. The file must follow the same format as in section 2.2, case 2, its name must be specified in `<Pi_NF>` and the file must be in the Inputs directory.

Note that the choice of `<Rdet>`=1 does not allow to choose the option `<Tsi>`=2 or `<Kex_type>`=4, to avoid any conflict between chemical and sediment parameters.

2.9 Flow supply for the flow concentration

We consider the flow supply given with the following equation:

$$F_{det} = \frac{P_i F_d (q - q_0)}{J_e} \left(1 - \left(\frac{\sum_i M_i}{M_{dt^*}}\right)\right), \quad \text{for} \quad t < TF_{end} \tag{7}$$

where F_d is the coefficient of the excess water flux, J_e the energy of entrainment, TF_{end} the depletion time of the flow supply and the other parameters are the same as previously. To set the flow supply, you should use `<Fdet>` as:

- case 0 to set the flow supply to zero `<Fdet>`=0.
- case 1 to set the flow supply with Equation (7). For this choice, specify the coefficients q_0, F_d, J_e and TF_{end} with `<q0>`, `<Fdcoef>`, `<Jecoef>` and `<TF_end>` respectively. You must also specify the parameters P_i and M_{dt^*} using `<Pi>` and `<Mscoef>` with the same principle as in the section 2.8, case 1.

Like for the rain supply, the choice of `<Fdet>`=1 cannot be used with `<Tsi>`=2 or `<Kex_type>`=4.

2.10 Chemical supply for the exchange layer

The chemical supply is calculated with the following equation:

$$Chem = SoilDiff \left.\frac{\partial C_s}{\partial z}\right|_{z=s/e} - I\left(\frac{M_i}{d_e}\right), \quad \text{for} \quad t < TC_{end} \tag{8}$$

where *SoilDiff* is the soil diffusion rate, s/e represents the interface between the soil and the exchange layer, I the infiltration, d_e the depth of the exchange layer, TC_{end} the depletion time of the chemical supply and C_s is the concentration in the soil, solution of $\frac{\partial(\alpha C_s)}{\partial t} = \frac{\partial}{\partial z}(SoilDiff \frac{\partial C_s}{\partial z} - IC_s)$. To set the chemical supply, you must use `<Chem>` as:

- case 0 to set the chemical supply to zero `<Chem>`=0.
- case 1 to set the chemical supply with Equation (8). For this choice, specify the parameters *SoilDiff*, d_e and TC_{end} using `<SoilDiff>`, `<de>` and `<TC_end>` respectively. The infiltration I is provided by the hydrodynamic part of FullSWOF_1D.

In the aim of dissociating sediment and chemical transfer, when `<Chem>`=1, the following choices cannot be used: `<Tsi>`=1, `<Kex_type>`=1, 2, 3, `<Rdet>`=1, `<Fdet>`=1.

2.11 Name of output directory

The results are saved in the directory named Outputs. Like for the hydrodynamic part of FullSWOF_1D, you can add a **suffix** to this name with `<suffix_t>` that can be different from the one of the hydrodynamic part.

3 Input values for the flow

If the Transfer equations are not coupled with the flow part of FullSWOF_1D, you must give characteristic values for the flow in the last part of the transfer.txt file. You have to fill the number of space cells `<Nxcell>` and the length of the domain `<L>` [m], the time step `<dt>` [s] and the number of time steps `<ndt>`. You must also give the values of the water height and the velocity; for the moment, the flow must be at steady-state, with the same constant value of the water height (and velocity) on the whole domain. These two values correspond to the `<H>` [m] and `<V>` [$\mathrm{m\,s^{-1}}$] tags respectively.

4 Outputs files

At the beginning of a run, the transfer code saves two files:

transfer.dat contains only the values used by the transfer code. This file has the same format as the transfer.txt file without the unnecessary parameters.

zcM_initial.dat contains the initial conditions for the topography z, the flow concentration c_i and the exchange layer concentration M_i for each class i.

During the computation, two files are created and incremented:

zcM_evolution.dat contains the evolution in time of the topography z, the concentrations c_i, M_i of each class i and the total concentrations $\sum_{i=1}^{N} c_i$, $\sum_{i=1}^{N} M_i$.

sediment_output_mass_evolution.dat contains the evolution in time of the mass of sediment [kg] that exited the domain, with the i^{th} class in the $i+1^{\text{th}}$ column. The last column contains the total mass, given by the sum over the classes.

For theses two evolution files, the number of times saved can be specified by using `<nbtimes_conc>`.

You can choose to create an additional file:

cM_temporal.dat contains the values at each time of the concentrations c_i, M_i of each class i and the total concentrations $\sum_{i=1}^{N} c_i$, $\sum_{i=1}^{N} M_i$ at a fixed x position. You can activate the creation of this file by choosing `<Temp>`=1 and specifying the position `<X_temp>`. If `<Temp>`=0, the file cM_temporal.dat will not be created.

At the final time, the transfer code creates two other files:

zcM_final.dat contains the values, on each cell, of the topography z, the concentrations c_i and M_i for each class i at the final time.

results_transfer.dat contains the sediment mass balance at the final time for each class, and a global sediment mass balance. More precisely, for each class i, 8 quantities are saved:

- Mass of sediment initially in the domain [kg] is the mass of sediment of class i in the domain at the beginning of the simulation.
- Mass of active sediment initially in the domain [kg] is the mass of sediment of class i in the domain where the mass of deposited sediment is multiplied by α, at the beginning of the simulation.
- Mass of sediment that entered [kg] is the mass of sediment of class i that entered the domain during the simulation.
- Mass of sediment that exited [kg] is the mass of sediment of class i that exited the domain during the simulation.
- Mass of active deposited sediment [kg] is α times the mass of sediment of class i that is still deposited over the bottom at the final time.
- Mass of transported sediment [kg] is the mass of sediment of class i that is still present in the water at the final time.
- Absolute error on the sediment mass conservation [dimensionless] expresses the numerical error of the theoretical mass conservation, which states that $hc_i + \alpha M_i$ must be constant in the absence of source terms S_1 and S_2. It is computed as the difference between the output mass and the input mass of sediment of class i during the run. The output mass is defined as the sum of the mass of transported sediment, the mass of deposited sediment multiplied by α, and the mass of sediment that exited. The input mass is defined as the sum of the mass of active sediment that entered and the mass of active sediment that were initially in the domain. The error is positive if sediment was created, and negative if sediment was lost during the run.
- Relative error on the sediment mass conservation [%] is the quotient of the absolute error by the input mass, multiplied by 100 to get a percentage. It is positive if sediment was created, and negative if sediment was lost during the run.

This file also contains these masses and errors for the total mass of sediment, corresponding to the sum over the classes.

5 Inputs parameters for some models

5.1 Hairsine and Rose (1991, 1992)

The model developed by Hairsine and Rose (1991, 1992) with the concept of detachment and redetachment can be re-written as:

$$\begin{cases} \dfrac{\partial hc_i}{\partial t} + \dfrac{\partial (qc_i)}{\partial x} = \dfrac{v_i}{h}\left(\left(\dfrac{F\sigma(\Omega-\Omega_0)}{v_i g(\sigma-\rho)M_{dt^*}} + \dfrac{a_d hR}{v_i M_{dt^*}}\right)M_i - hc_i\right) + (r_{ei}+e_i) \\ \dfrac{\partial M_i}{\partial t} = -\dfrac{v_i}{h}\left(\left(\dfrac{F\sigma(\Omega-\Omega_0)}{v_i g(\sigma-\rho)M_{dt^*}} + \dfrac{a_d hR}{v_i M_{dt^*}}\right)M_i - hc_i\right) \end{cases} \tag{9}$$

where c_i is the sediment concentration in size class i and v_i the settling velocity. The stream power per unit bed area Ω is defined as: $\rho g S q$ with ρ the water density, g the acceleration due to gravity, S the slope and q the water flux per unit width. The stream power threshold is Ω_0 ($=\rho g S q_0$) and the effective fraction of $\Omega - \Omega_0$ is denoted $F(\Omega - \Omega_0)$. The coefficient σ is the sediment density, ρ the water density, M_{dt^*} the mass for complete shielding, a_d the detachability of the deposited layer and R the rain intensity. The total number of sediment classes is N and the rate of flow entrainment r_{ei} is defined as: $r_{ei} = \frac{F}{NJ}(\Omega - \Omega_0)(1 - \sum_{i=1}^{N} M_i/M_{dt^*})$ with J the energy per unit mass of soil of entrainment. The rainfall detachment e_i is calculated with: $e_i = \frac{a_i R}{N}(1 - \sum_{i=1}^{N} M_i/M_{dt^*})$ with a_i the detachability of the initial soil.

The identification of the different parameters of the Transfer equation (1) gives:

$$ts_i = \frac{h}{v_i} \tag{10a}$$

$$g(M_i) = (Ke_i + Kr_i)M_i \tag{10b}$$

$$Ke_i = \frac{a_d}{M_{dt^*}} ts_i R \tag{10c}$$

$$Kr_i = \frac{F\sigma(\Omega-\Omega_0)}{v_i g(\sigma-\rho)M_{dt^*}} = \frac{F\sigma\rho S}{v_i(\sigma-\rho)M_{dt^*}}(q-q_0) \tag{10d}$$

$$S_1 = S_D + S_F \tag{10e}$$

$$S_D = e_i = \frac{a_i R}{N}\left(1 - \frac{\sum_{i=1}^{N} M_i}{M_{dt^*}}\right) \tag{10f}$$

$$S_F = r_{ei} = \frac{F\rho g S}{NJ}(q-q_0)\left(1 - \frac{\sum_{i=1}^{N} M_i}{M_{dt^*}}\right) \tag{10g}$$

$$S_2 = 0 \tag{10h}$$

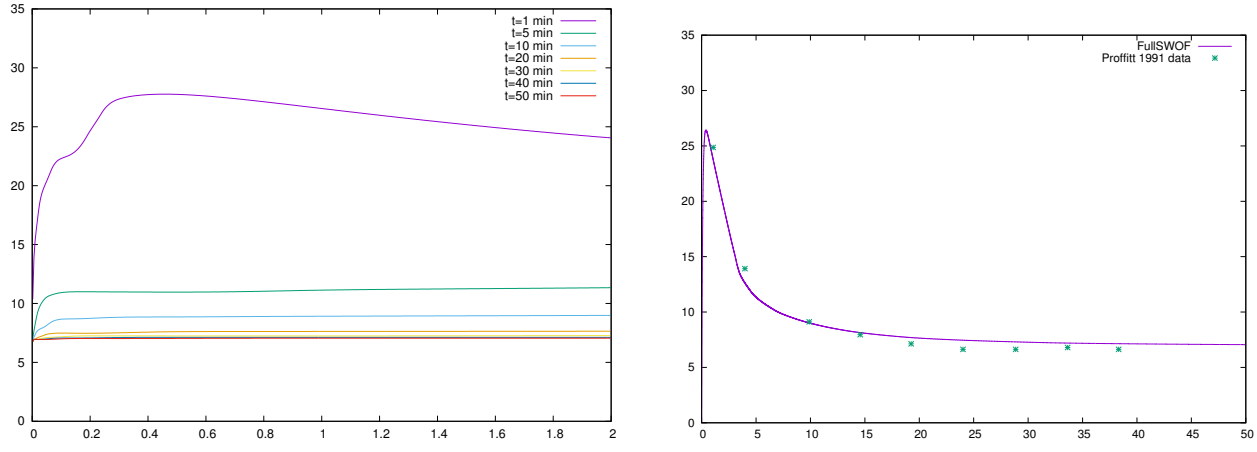
In the file transfer.txt, you can use the values contained in Table 1 (see other parameters corresponding to the numerical solution of Hogarth et al. (2004a) in the Examples/Rainfall_detachment directory)

Table 1: Inputs parameters for the model of Hairsine and Rose (1991, 1992).

Parameter	**Value**
Number of classes	
`<Nc>`	N
Coefficient of the concentration M_i	
`<Alphacoef>`	1
Relaxation time	
`<Rpi>`	2
`<Rpi_NF>`	contains the value of $1/v_i$
`<Tsi>`	1
Equilibrium function	

<GMi>	1
<Kex_type>	3
<ki>	1
<kicoef>	a_d/M_{dt^*}
<ki2>	2
<Ki_NF2>	contains the value of $\frac{F\sigma\rho S}{v_i(\sigma-\rho)M_{dt^*}}$
<R0>	0
<q0>	q_0
Rain supply	
<Rdet>	1
<Adcoef>	a_i
<B>	0
<Bcoef>	1
<TR_end>	final time
Flow supply	
<Fdet>	1
<Fdcoef>	$F\rho gS$
<Jecoef>	J
<TF_end>	final time
<Mscoef>	M_{dt^*}
<Pi>	1
<Picoef>	$1/I$

As explained in Nouhou Bako et al. (2021), we follow Hogarth et al. (2004b) in order to numerically reproduce the experiments of Proffitt et al. (1991): they model rainfall-driven erosion over a slope $S = 0.4\%$ considering $N = 10$ classes of particles uniformly distributed, with a constant rain of $R = 100\,\mathrm{mm\,h^{-1}}$. They propose a numerical model and they fit the values of the detachabilities a and a_d and of the mass of deposited particles to shield the original soil M_{dt^*}. As their numerical solution is in good agreement with the experiments, we take the same parameters as the ones they use. They found the value of M_{dt^*} to be $M_{dt^*} = 0.0767\,\mathrm{kg\,m^{-2}}$: M_{dt^*} is larger than M_t (so $H = \frac{M_t}{M_{dt^*}}$), such that G becomes a linear function. As they also get an approximation of the water height as a linear function, we choose to initialize our flow solver with this expression and let it evolve in time towards the equilibrium.



(a) Concentration for different times

(b) Concentration at the output

Figure 1: Comparison of the experimental data of Proffitt et al. (1991) and the numerical resolution of the transfer equations by FullSWOF_1D-transfer

In Figure 1, we plot the total concentration of particles detached by the rain and suspended in the fluid as a function of the space for several times (Figure 1a), and, at the output, as a function of

time (Figure 1b). We start from flow conditions near the equilibrium, and the steady state is attained after some time, solving the Shallow Water equations coupled with the Hairsine and Rose equations for the particles.

5.2 Lajeunesse et al. (2013)

Lajeunesse et al. (2013) describe the evolution of the proportion of marked grains in a moving layer $C = hc$ and on the bed surface M with the following model:

$$\begin{cases} \dfrac{\partial C}{\partial t} + \dfrac{\partial (uC)}{\partial x} = \dfrac{1}{ts}(M - C) \\ \dfrac{\partial M}{\partial t} = -\dfrac{\beta}{ts}(M - C) \end{cases} \tag{11}$$

where u is the flow velocity, ts the characteristic settling time of a particle and β the ratio of the concentration of moving grains to the concentration of static grains. In this model, the parameters of the Transfer equation (1) are:

$$ts = ts \tag{12a}$$
$$g(M) = M \tag{12b}$$
$$\alpha = \frac{1}{\beta} \tag{12c}$$
$$S_1 = 0 \tag{12d}$$
$$S_2 = 0. \tag{12e}$$

In the file transfer.txt, you can use the values contained in Table 2 (see other parameters in the directory Benchmarks/Lajeunesse_bedload_transport_phipsi).

Table 2: Inputs parameters for the model of Lajeunesse et al. (2013)

Parameter	**Value**
Number of classes	
`<Nc>`	1
Coefficient of the concentration M_i	
`<Alphacoef>`	$1/\beta$
Relaxation time	
`<Rpi>`	1
`<Rpicoef>`	ts
`<Tsi>`	0
Equilibrium function	
`<GMi>`	1
`<Kex_type>`	0
`<ki>`	1
`<kicoef>`	1

Note that this model can also be written, up to a change of variables detailed in Nouhou Bako et al. (2021), under the following form:

$$\begin{cases} \dfrac{\partial C}{\partial t} + \dfrac{\partial (uC)}{\partial x} = \dfrac{1}{ts}(\beta M - C) \\ \dfrac{\partial M}{\partial t} = -\dfrac{1}{ts}(\beta M - C). \end{cases} \tag{13}$$

Under this form, the parameters of the Transfer equation (1) are:

$$ts = ts \tag{14a}$$
$$g(M) = \beta M \tag{14b}$$
$$\alpha = 1 \tag{14c}$$
$$S_1 = 0 \tag{14d}$$
$$S_2 = 0. \tag{14e}$$

If you want to solve the bedlload transport problem with this other writing of the system, in the file transfer.txt, you should use the values contained in Table 3 (see other parameters in the directory Benchmarks/Lajeunesse_bedload_transport_cM). The differences between these parameters and the previous set of parameters in table 2 are the values of `<Alphacoef>` and `<kicoef>`.

Table 3: Inputs parameters for the model of Lajeunesse et al. (2013)

Parameter	**Value**
Number of classes	
`<Nc>`	1
Coefficient of the concentration M_i	
`<Alphacoef>`	1
Relaxation time	
`<Rpi>`	1
`<Rpicoef>`	ts
`<Tsi>`	0
Equilibrium function	
`<GMi>`	1
`<Kex_type>`	0
`<ki>`	1
`<kicoef>`	β

5.3 Gao et al. (2004)

In a context of chemical transfer, Gao et al. (2004) present a model for the chemical transport induced by rainfall. Their equations are arranged as follows:

$$\begin{cases} \dfrac{\partial hc}{\partial t} + \dfrac{\partial qc}{\partial x} = \dfrac{\lambda e_r + I}{h}\left(\left(\dfrac{e_r h}{d_e(\lambda e_r + I)}\right) M - hc\right) \\ \alpha \dfrac{\partial M}{\partial t} = -\dfrac{\lambda e_r + I}{h}\left(\left(\dfrac{e_r h}{d_e(\lambda e_r + I)}\right) M - hc\right) + \mathit{SoilDiff}\ \dfrac{\partial C_s}{\partial z}\Big|_{z=s/e} - I\dfrac{M}{d_e}. \end{cases} \tag{15}$$

In Equation (15), λ is the fraction of the concentration c entering the exchange layer, $e_r = a\theta R/\rho_b$ is the raindrop induced water transfer with a the soil detachability, θ the soil water content, ρ_b the soil dry bulk density and R the rain intensity. The depth of the exchange layer is denoted d_e, the infiltration I and *SoilDiff* is the diffusion rate of the soil assumed here to be constant. The quantity αM is the proportion of M involved in exchange process. For this model, the parameters of the Transfer equation (1) are:

$$ts = \left(\frac{\lambda e_r + I}{h}\right)^{-1} = \left(\frac{\frac{\lambda a\theta}{\rho_b}R + I}{h}\right)^{-1} \tag{16a}$$

$$g(M) = KM, \qquad K = \frac{e_r h}{d_e(\lambda e_r + I)} \tag{16b}$$

$$S_1 = 0 \tag{16c}$$

$$S_2 = SoilDiff\ \frac{\partial C_s}{\partial z}\bigg|_{z=s/e} - I\frac{M}{d_e} \quad \text{with} \quad \frac{\partial(\alpha C_s)}{\partial t} = \frac{\partial}{\partial z}(SoilDiff\frac{\partial C_s}{\partial z} - IC_s) \tag{16d}$$

Table 4 contains the values of the input parameters that can be used in the file transfer.txt (see the Examples/Chemical_transport directory for other parameters).

Table 4: Inputs parameters for the model of Gao et al. (2004)

Parameter	**Value**
Number of classes	
`<Nc>`	1
Coefficient of the concentration M_i	
`<Alphacoef>`	α
Relaxation time	
`<Rpi>`	1
`<Rpicoef>`	$\lambda a\theta/\rho_b$
`<Tsi>`	2
`<R0_chem>`	0
Equilibrium function	
`<GMi>`	1
`<Kex_type>`	4
`<ki>`	1
`<kicoef>`	$a\theta/d_e\rho_b$
Chemical supply	
`<Chem>`	1
`<SoilDiff>`	*SoilDiff*
`<de>`	d_e
`<TC_end>`	final time

As detailed in Nouhou Bako et al. (2021), we reproduce the first experiment of Gao et al. (2004), taking their values of the parameters, namely a rain intensity of $7.4\,\mathrm{cm\,h^{-1}}$, an initial concentration of Cl^- in the soil and in the exchange layer of $29.82\,\mathrm{g\,L^{-1}}$, a water height of 7 mm, a height of the exchange layer of 7.6 mm, a bulk density of $\rho_b = 1.5\,\mathrm{g\,cm^{-1}}$, a soil moisture of $\theta = 0.37 = \alpha$, a soil erodibility of $a = 0.40\,\mathrm{g\,cm^{-3}}$, a soil diffusivity coefficient of $4.2 \times 10^{-6}\,\mathrm{cm^2\,s^{-1}}$, a parameter $\lambda = 1$, and a raindrop chemical transfer rate of $e_r = 2.1 \times 10^{-4}\,\mathrm{cm\,s^{-1}}$.

In Figure 2, we plot the solute concentration (i.e. Cl^-) in the exchange layer and in the runoff layer corresponding to this experiment with the numerical approximation proposed by Gao et al. (2004) represented as cross marks. FullSWOF_1D-transfer solves the Shallow-Water equations and the transfer equations with the same parameters, and we choose a point of the domain to plot the evolution in time. In both layers, the concentrations as computed by the unifying equations match very well the results of Gao et al. (2004).

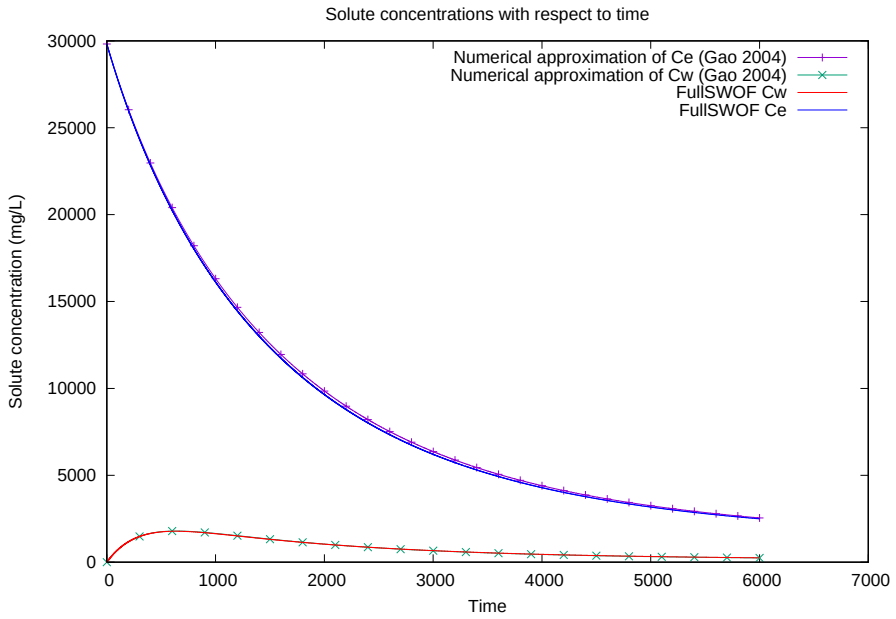


Figure 2: Numerical approximations of the solute concentrations in the exchange layer (C_e) and in the ponding water layer (C_w) proposed by Gao et al. (2004), and the numerical results of FullSWOF_1D-transfer at $x = 0.025\,\mathrm{m}$.

6 Validation

6.1 Two classes of sediment eroded and deposited by the water

Before doing comparisons with physical models, we perform a test to check the proper functioning of the software. This is possible because we were able to write a nearly-exact solution at each time and at each point, for a given choice of parameters. Each software computing the solutions of the transfer equations (1) should return a good approximation of this solution under the corresponding conditions, and the results should be improved when taking a smaller space step.

We consider two sizes of particles inside the domain, distributed in the flowing and the exchange layers, without any source, and we let the system evolve in time. We choose $G(M_i) = K_i M_i$, with $K_1 = 243$ and $K_2 = 0.3$, $ts_1 = 0.087\,\mathrm{s}$, $ts_2 = 7.17 \times 10^{-4}\,\mathrm{s}$ and $A = 1$. We solve the system

$$\begin{cases} \dfrac{\partial(hc_1)}{\partial t} + \dfrac{\partial(qc_1)}{\partial x} = \dfrac{1}{0.087}(243M_1 - hc_1) \\ \dfrac{\partial M_1}{\partial t} = -\dfrac{1}{0.087}(243M_1 - hc_1), \\ \dfrac{\partial(hc_2)}{\partial t} + \dfrac{\partial(qc_2)}{\partial x} = \dfrac{1}{0.000717}(0.3M_2 - hc_2) \\ \dfrac{\partial M_2}{\partial t} = -\dfrac{1}{0.000717}(0.3M_2 - hc_2), \end{cases}$$

starting from $h(t = 0, x) = 3.75 \times 10^{-3}\,\mathrm{m}$ and $u(t = 0, x) = 0.3391\,\mathrm{m\,s^{-1}}$ to be at a steady state for the water over an inclined plane with slope -0.02 (the velocity u is given by $u = q/h$ and stays constant as well as the water height), and

$$c_1(t = 0, x) = \begin{cases} x - 1 & \text{for } 1 \le x \le 2 \\ 1 & \text{for } 2 \le x \le 3 \\ 4 - x & \text{for } 3 \le x \le 4 \\ 0 & \text{elsewhere,} \end{cases} \quad c_2(t = 0, x) = \begin{cases} x - 1.5 & \text{for } 1.5 \le x \le 2 \\ 0.5 & \text{for } 2 \le x \le 3 \\ 3.5 - x & \text{for } 3 \le x \le 3.5 \\ 0 & \text{elsewhere,} \end{cases}$$

and $M_1(t = 0, x) = hc_1(t = 0, x)/K_1, M_2(t = 0, x) = hc_2(t = 0, x)/K_2$ for the particle concentrations (figure 3).

Summing the two equations for the same class of particles i, the right parts cancel two by two, and we get

$$\frac{\partial(hc_i + M_i)}{\partial t} + \frac{\partial(qc_i)}{\partial x} = 0.$$

In our configuration, as the ts_i are small numbers, using an asymptotic development, one can prove that a good approximation of the solution is given by $M_i(t,x) \approx hc_i(t,x)/K_i$ and, consequently, $c_i(t,x) \approx c_i\left(t = 0, x - \frac{q/h}{1+K_i}t\right)$. These theoretical approximations are useful to validate the code and to check the propagation velocity of the particles.

In Figure 3, we compare the values of c and M for the two classes of particles with the numerical solution obtained by FullSWOF_1D-transfer (see the parameters file in the Examples/Sediment_deposition folder in the repository). The results are given at time $T = 8\,\mathrm{s}$ and the values of M_1 are multiplied by 100 in order to see the curves within the same graph as M_2. The approximations of the exact solution and of the numerical solution are in good agreement. Note that we took 1000 cells in space (the space step is equal to 0.01) to have a good precision. If the number of cells is decreased, the results get less accurate, and the numerical errors accumulate with time.

6.2 Lajeunesse et al. (2013)

The second example of section 5 can be considered as benchmarks as we can compute an analytic solution. This benchmark is configured in the Benchmarks directory and we give here the results obtained with FullSWOF_1D-transfer.

For a constant flow velocity $u(t,x) = V$, equation (11) can be written in a dimensionless form

$$\begin{cases} \dfrac{\partial C}{\partial \widetilde{t}} + \dfrac{\partial(C)}{\partial \widetilde{x}} = M - C \\ \dfrac{\partial M}{\partial \widetilde{t}} = -\beta(M - C), \end{cases} \tag{17}$$

where $\widetilde{t} = t/ts$, $\widetilde{x} = x/(Vts)$. A numerical solution of this system can be computed, see Lajeunesse et al. (2013, Appendix A). For very small time and space steps, it can be considered as the analytic solution.

The initial proportions of marked grains are:

$$\begin{cases} M(0,\widetilde{x}) = \mathbb{1}_{]0,20]}(\tilde{x}), \\ C(0,\widetilde{x}) = 0. \end{cases}$$

In Figure 4, we compare the concentrations of marked grains in the fluid and on the bed surface, computed by FullSWOF_1D-transfer solving sytem (11) and given by the numerical solution of Lajeunesse et al. (2013). In the moving layer, the concentration is $\frac{\beta}{1+\beta}C$ and on the bed surface, the concentration is $\frac{1}{1+\beta}M$. The computations were performed with $\beta = 0.3$. FullSWOF_1D-transfer is run with $h = 1\,\mathrm{m}$ and $u = 1\,\mathrm{m\,s^{-1}}$ such that $x = \tilde{x}$, $t = \tilde{t}$ and $ts = 1$.

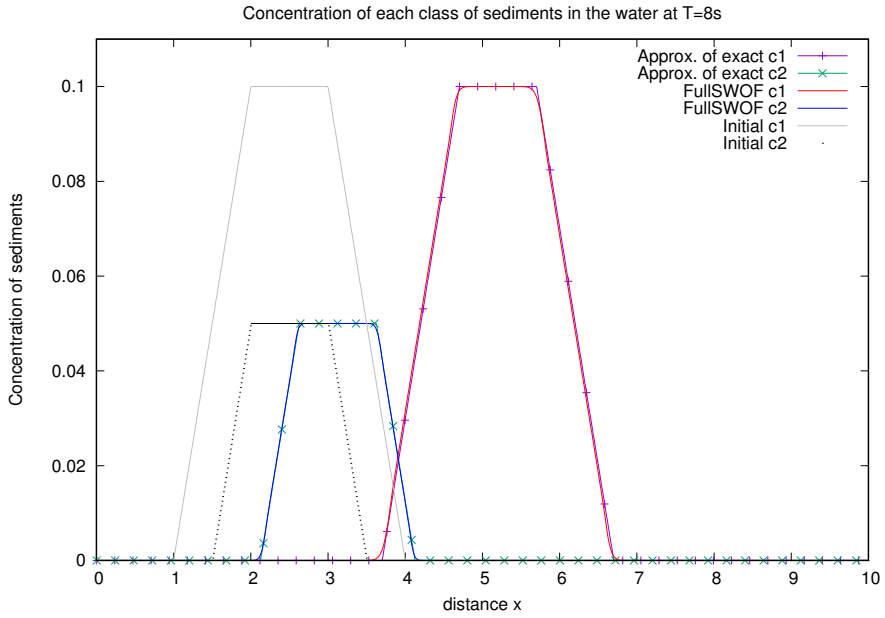
Similar results can be obtained solving system (13).

References

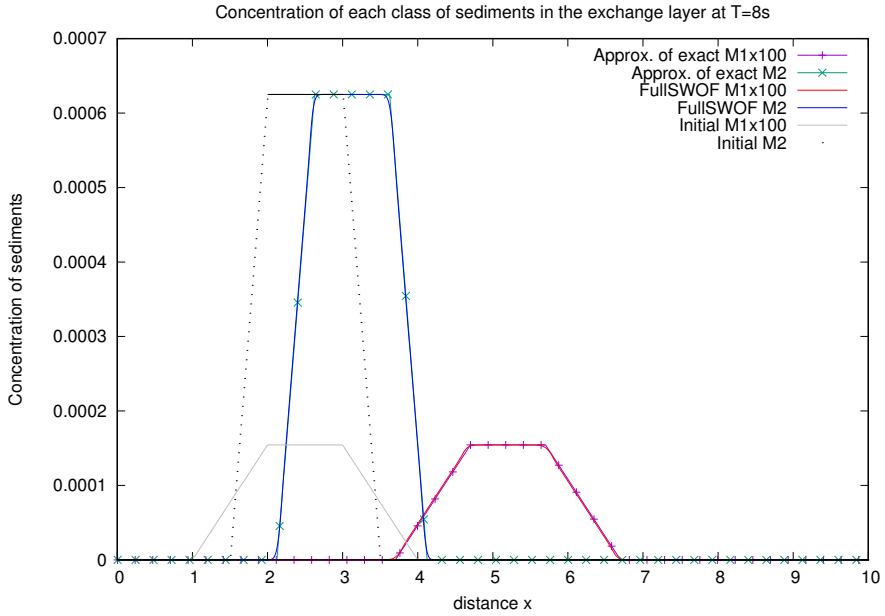
Gao, B., Walter, M. T., Steenhuis, T. S., Hogarth, W. L., and Parlange, J.-Y. (2004). Rainfall induced chemical transport from soil to runoff: Theory and experiments. *Journal of Hydrology*, 295(1-4):291–304, DOI: 10.1016/j.jhydrol.2004.03.026.

Hairsine, P. B. and Rose, C. W. (1991). Rainfall detachment and deposition: Sediment transport in the absence of flow-driven processes. *Soil Science Society of America Journal*, 55(2):320–324, DOI: 10.2136/sssaj1991.03615995005500020003x.

Hairsine, P. B. and Rose, C. W. (1992). Modeling water erosion due to overland flow using physical principles: 1. sheet flow. *Water Resources Research*, 28(1):237–243, DOI: 10.1029/91WR02380.



(a) Concentrations in the water



(b) Concentration in the exchange layer

Figure 3: Results for two classes of particles without sources evolving by erosion and deposition.

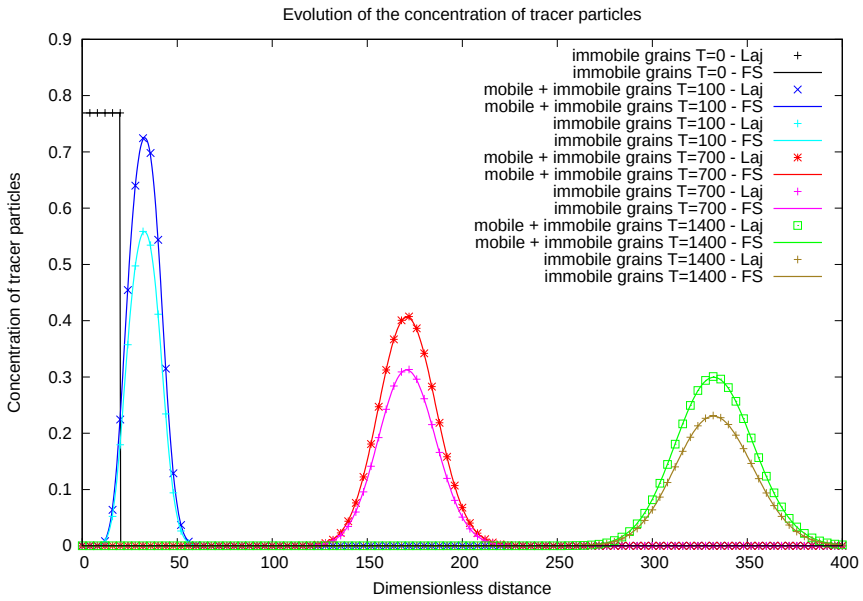


Figure 4: Evolution of the concentration of tracer particles

Hogarth, W. L., Parlange, J. Y., Rose, C. W., Sander, G. C., Steenhuis, T. S., and Barry, A. (2004a). Soil erosion due to rainfall impact with inflow: an analytical solution with spatial and temporal effects. *Journal of Hydrology*, 295(1-4):140–148, DOI: 10.1016/j.jhydrol.2004.03.007.

Hogarth, W. L., Rose, C. W., Parlange, J. Y., Sander, G. C., and Carey, G. (2004b). Soil erosion due to rainfall impact with no inflow: a numerical solution with spatial and temporal effects of sediment settling velocity characteristics. *Journal of Hydrology*, 294:229–240, DOI: 10.1016/j.jhydrol.2004.02.014.

Lajeunesse, E., Devauchelle, O., Houssais, M., and Seizilles, G. (2013). Tracer dispersion in bedload transport. *Advances in Geosciences*, 37:1–6, DOI: 10.5194/adgeo-37-1-2013.

Nouhou Bako, A., Lucas, C., Darboux, F., James, F., and Gaveau, N. (2021). A unifying model for soil erosion, river bedload and chemical transport. Submitted.

Proffitt, A. P. B., Rose, C. W., and Hairsine, P. B. (1991). Rainfall detachment and deposition: Experiments with low slopes and significant water depths. *Soil Science Society of America Journal*, 55(2):325–332, DOI: 10.2136/sssaj1991.03615995005500020004x.