Laminar friction law
and validation with self-similar solutions for dam breaks.

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In this section, we explain the laminar friction law and we give some comparisons with self-similar solutions for dam breaks.

1 Model and integration in *FullSWOF_1D*

The Shallow-Water equations read[1]:

$$\begin{cases} \partial_t h + \partial_x q = 0, \\ \partial_t q + \partial_x \left(\dfrac{q^2}{h} + \dfrac{g}{2} h^2 \right) = -3\nu \dfrac{q}{h^2} = -gh \dfrac{3\nu}{gh} \dfrac{q}{h^2}, \end{cases} \tag{1}$$

where h is the water height, q the discharge, g the gravity and 3ν the laminar friction coefficient, where ν is the kinematic viscosity. Then (using the notations of [3]), the laminar friction term is $S_f = \dfrac{3\nu}{gh} \dfrac{q}{h^2}$.

As in [3], at the first order, after a convective step $W_i^* = W_i^n - \Delta t \Phi(W^n)$, the value of W_i^{n+1} is given by:

$$W_i^{n+1} = \begin{pmatrix} h_i^{n+1} \\ q_i^{n+1} \end{pmatrix} = \begin{pmatrix} h_i^* \\ q_i^* \left(1 + 3\nu \dfrac{\Delta t}{\left(h_i^{n+1}\right)^2} \right)^{-1} \end{pmatrix}.$$

This law has been validated thanks to two self-similar solutions programmed in *SWASHES*.

2 Self-similar solution on a flat bottom

This solution is obtained in [1, section 5.2] and corresponds to the solution of the diffusive wave approximation. At the initial time, the water height is $h(t=0) = 0.4 \times \mathbb{1}_{[7.5,12.5]}$, the velocity is null. The topography is flat, the friction coefficient is $3\nu = 0.3$ m^2/s, the final time is $T = 30$ s. We plot on Figure 1 the analytic self-similar solution obtain in [1] and computed by *SWASHES* and the results of *FullSWOF_1D* for several space steps. The computed solution converges to the self-similar solution.

[1] note that, with the laminar friction law, the momentum equation should read :

$$\partial_t q + \partial_x \left(\frac{6}{5} \frac{q^2}{h} + \frac{g}{2} h^2 \right) = -3\nu \frac{q}{h^2}.$$

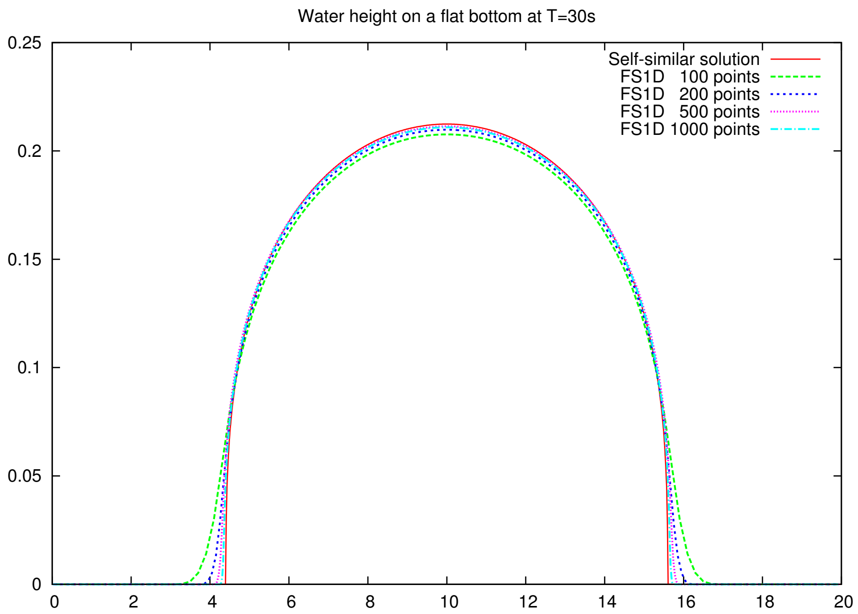

Figure 1: Water height on a flat bottom

3 Self-similar solution on an inclined plane

We also compare the solutions on an inclined plane $z_b = -0.1x + 3$, obtained in [2] and [1, section 5.1]. In that case, the self-similar solution corresponds to the kinematic wave approximation, and the profile ends abruptly (which is not realistic). The solution has not been smoothed off thanks to the surface tension, which means that *FullSWOF_1D* will not converge exactly to the self-similar solution.

At the initial time, the water height is $h(t = 0) = 0.1 \times \mathbb{1}_{[2,12]}$, the velocity is null. The friction coefficient is $3\nu = 0.3$ m^2/s, the final time is $T = 100$ s. We plot on Figure 2 the analytic unsmoothed self-similar solution obtain in [1] and computed by *SWASHES* and the results of *FullSWOF_1D* for several space steps. The computed solution has converged to a solution that is not exactly the self-similar solution, as expected.

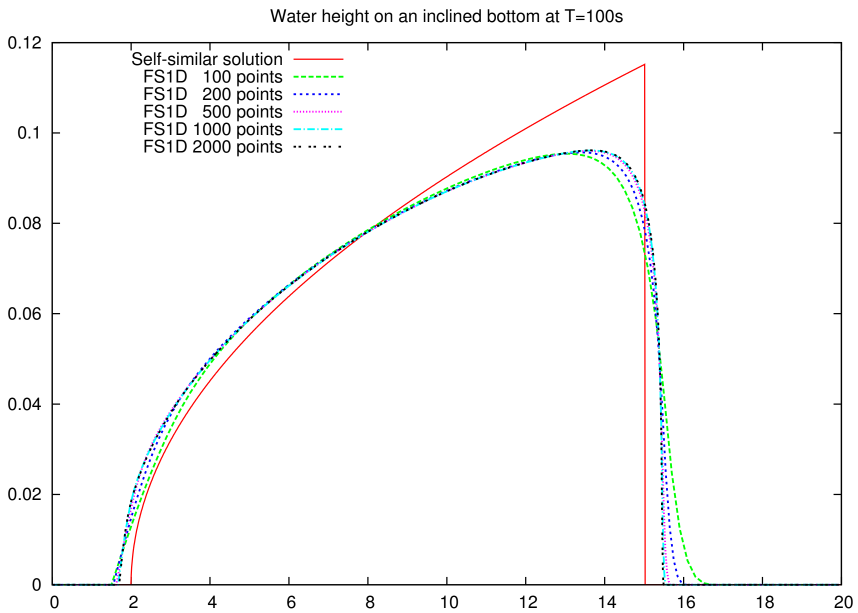

Figure 2: Water height on an inclined bottom

References

[1] Serge Hèzièwè Bodjona, Etude d'exemples simples d'écoulements, comparaison Saint-Venant et Navier-Stokes. *Master's thesis*, Mécanique des Fluides, Fondements et Applications, Université P. & M. Curie, France, 2013.

[2] Herbert E. Huppert. Flow and instability of a viscous current down a slope. *Nature*, 300:427–429, 1982. doi:10.1038/300427a0.

[3] Olivier Delestre, Frédéric Darboux, Francois James, Carine Lucas, Christian Laguerre, and Stephane Cordier. FullSWOF: A free software package for the simulation of shallow water flows. Preprint, 2014. URL `http://hal.archives-ouvertes.fr/hal-00932234`.