Comparison between *FullSWOF_2D* with a fixed time step and a fixed CFL number. v0.00.01

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The purpose of this report is to compare the running of *FullSWOF_2D* with either fixed CFL number or fixed time step.

The benchmarks used to this comparison are:
-Lake at rest with emerged bump,
-Dam break Dry without friction,
-MacDo Pseudo 2D in the long channel with subcritical flow and Manning's friction,
-MacDo Pseudo 2D in the short channel with subcritical flow and Manning's friction,
-MacDo Rain long in the channel with subcritical flow Darcy-Weisbach's friction,
-MacDo Rain long in the channel with supercritical flow Darcy-Weisbach's friction,
-MacDo on the Short channel smooth transition shock,
-Thacker planar surface in paraboloid.

1 Choice of time step

In this section, we describe the method to pick out the time step for the simulation with fixed time step.
In fact, I have run *FullSWOF_2D* with the inputs of the previous benchmarks and I have chosen the time step which has the smaller value.
All simutations have been run with a CFL number equal to 0,5.
The results are plotted on the figures 1, 2, 3, 4, 5, 6.

I haven't plotted the bump and dam break's results because for these two benchmarks the time step was constant during the simulation with fixed CFL number, so I took this value of time step to run the simulation with fixed time step.
In case of Bump's simulation $dt = 0.1s$ and dam break $dt = 0.04s$.

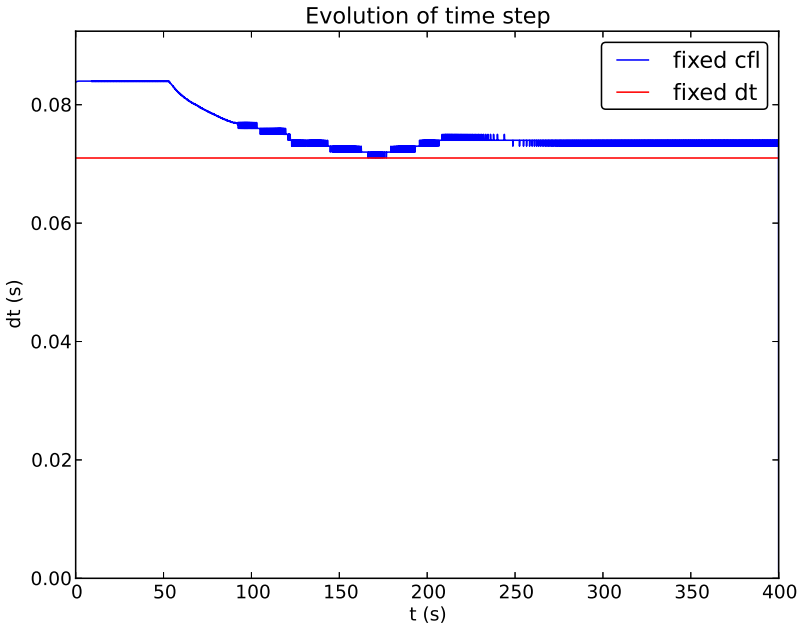

Figure 1: MacDo Pseudo 2D long channel subcritical flow Manning's friction

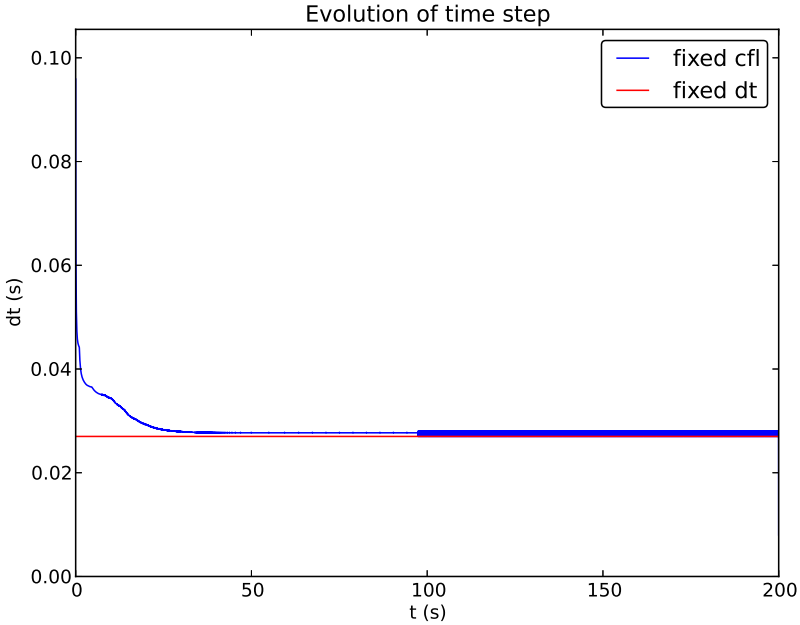

Figure 2: MacDo Pseudo 2D short channel supercritical flow Manning's friction

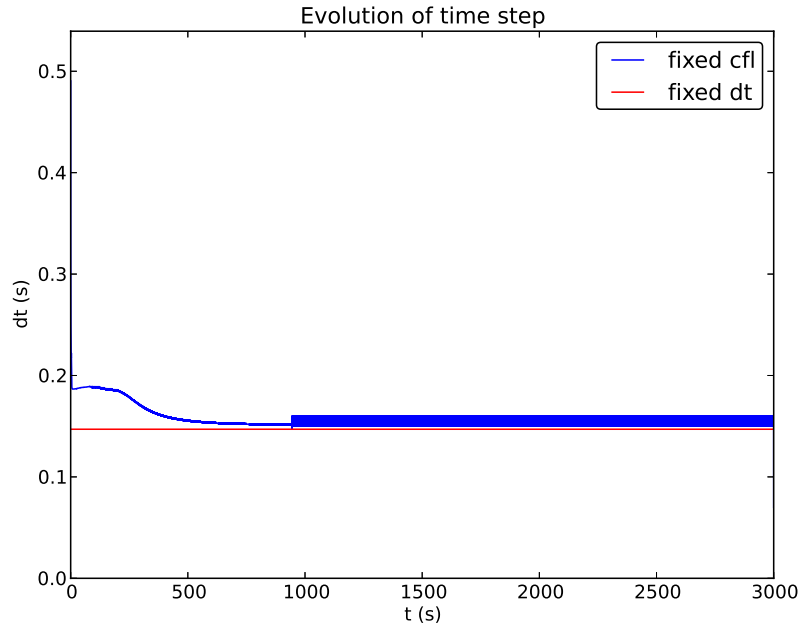

Figure 3: MacDo Rain long channel subcritical flow DW

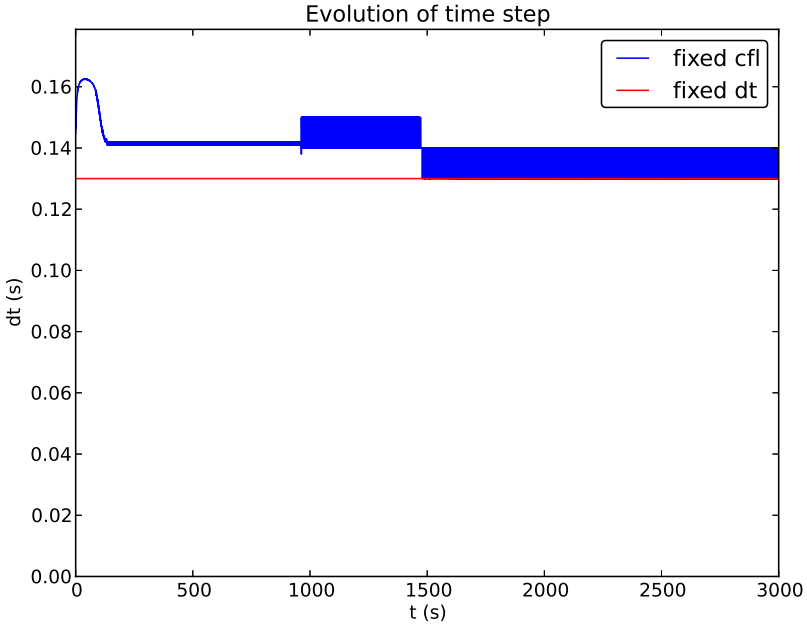

Figure 4: MacDo Rain long channel supercritical flow DW

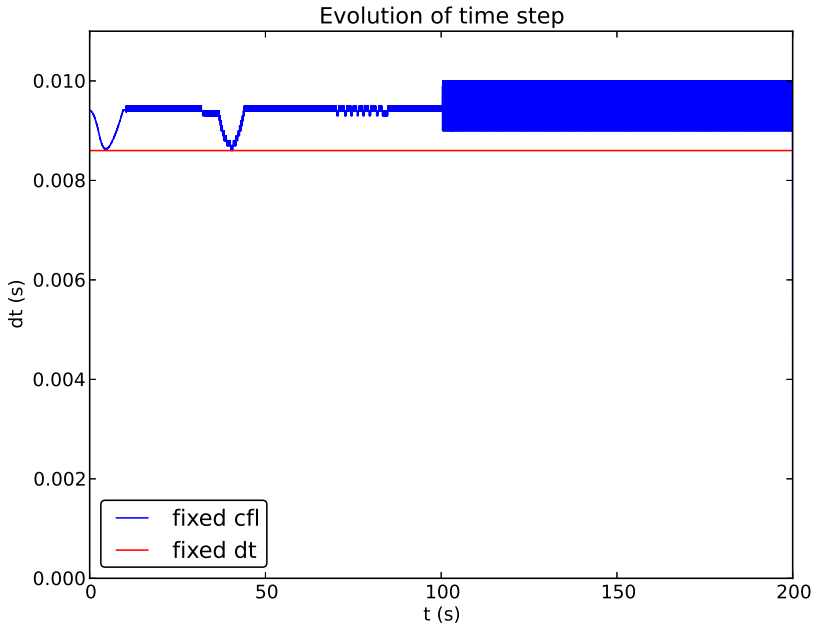

Figure 5: MacDo Short channel smooth transition shock

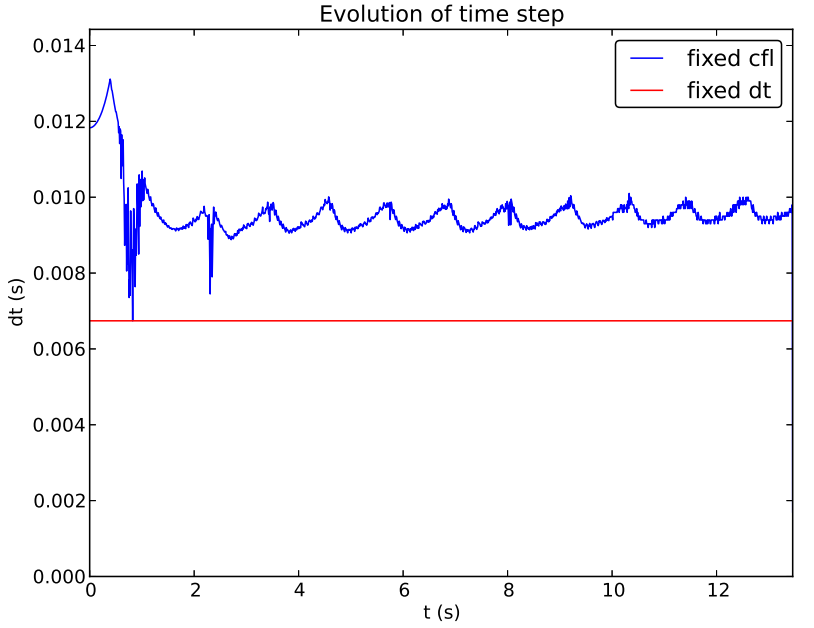

Figure 6: Thacker planar surface in paroboloid

Table 1: The Results of comparison.

	error dt/exact (%)	error cfl/exact (%)	error dt/cfl (%)	CPU time (s): dt	CPU time (s): cfl
Bump	0	0	0	24	25
Dam break	4.5	4.5	0	4	3
Pseudo 2D sub	7.8	7.8	3.e-03	86	128
Pseudo 2D sup	1.3	1.3	0	116	125
Rain sub DW	0.2	0.2	0	159	244
Rain sup DW	5.e-03	5.e-03	7.e-05	329	484
Short channel	0.4	0.4	5.e-03	110	147
Thacker	0.7	3.5	3	41	44
Total				869 ($\approx$ 14mn)	1200 ($\approx$ 20mn)

2 Numerical results

In this section, I will present the results obtained for the water height and the velocity in comparison with analytic solution.
To compare the different configurations (i.e *FullSWOF_2D* ran with a fixed time step or fixed CFL number), I calculated two errors for each simulation with a L1's norm.
The first error (error dt/exact or error cfl/exact) is used in order to know the difference between the analytic solution and the simulation with either fixed time step or CFL's number. The calcul applied for this is: $error = \frac{\|h_a - h\|}{\|h_a\|}$,
where h_a is the analytic solution water height and h is the water height obtained from one of both simulations.
The second error (error dt/cfl) is used to compare the difference between the two configuration. Hence, the formula applied is: $error = \frac{\|h_{cfl} - h_{dt}\|}{\|h_{cfl}\|}$,
where h_{cfl} is the water height from simulation with CFL's number and h_{dt} is the water height from simulation with a fixed time step.
We give a table (see table 1) which sum up the results obtained for the different simulations.
Thus, as table 1 shows, the simulation with a fixed time step executes all benchmarks, with a better accuracy, more faster than the simulation with a CFL's number. The greater diffecence in term of accuracy is to the running of Thacker's benchmark (Figure 7). In this case, the simulation with a fixed time step is better than the simulation with a fixed CFL's number. Indeed, for this simulation the relative error is equal to 0.7% while the error is equal to 3% in case of the simulation with CFL's number.

3 Conclusion

The conclusion is when we run *FullSWOF_2D* with a fixed time step, the error are smaller than *FullSWOF_2D* with a CFL number. Moreover, *FullSWOF_2D*

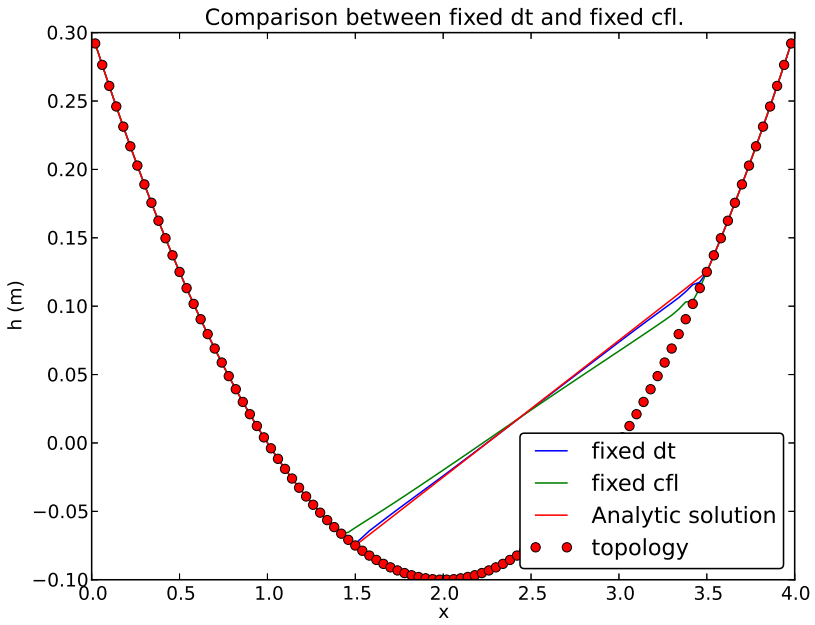

Figure 7: Thacker planar surface in paroboloid water height.

consumes less CPU time when it runs with a fixed time step.
So, if you have a assessment of the time step's value, you would take advantage of running *FullSWOF_2D* with a fixed time step.
You can also decide to run *FullSWOF_2D* with a CFL number to have a estimate of time step and after running it with a fixed time step.

4 Appendix

In this appendix, we describe the parameters of the benchmarks.

4.1 Lake at rest with emerged bump

PARAMETERS OF THE SOLUTION
Length of the domain: 25 meters
Width of the domain: 25 meters
Space step along x: 0.25 meters (Nb mailles:100)
Space step along y: 0.25 meters (Nb mailles:100)
Topography: $z(x) = max(0.0, 0.2 - 0.05 * (x - 10)^2)$
Solution at the steady state

Initial condition: h+z = 0.1 m and q = 0 m^2/s
Imposed discharge (left-inflow) $q_{in} = 0$ m^2/s
Imposed water height (right-outflow) $h_{out} = 0.1$ m

Bump solution - steady state at rest with an emerged bump

4.2 Short channel with smooth transition and shock

PARAMETERS OF THE SOLUTION
Length of the domain: 100 meters
Width of the domain: 1 meters
Space step along x: 0.2 meters
Space step along y: 0.2 meters
Solution at the steady state

Initial conditions: $h = max(2.87871 + z(100) - z, 0)$ m and $q = 0\ m^2/s$
Imposed discharge on the left boundary: $2\ m^2/s$
Imposed water height on the right boundary: $2.87871m$
Manning's friction coefficient: 0.0328

MacDonald's solution - subcritical-subcritical with Manning law

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4.3 Rain on a long channel with subcritical flow

PARAMETERS OF THE SOLUTION
Length of the domain: 1000 meters
Width of the domain: 10 meters
Space step along x: 2 meters
Space step along y: 2 meters
Rain intensity for t >= 1500 s: $0.001\ m/s$
Final time: 3000 s

Initial conditions: h = 0 m and q = $0\ m^2/s$
Imposed discharge on the left boundary for the first iteration: $1\ m^2/s$
Imposed water height on the left boundary: 0.748324 m
Imposed water height on the right boundary: 0.748324 m
Darcy-Weisbach's friction coefficient: 0.093

MacDonald's solution - subcritical-subcritical with rain with Darcy-Weisbach law

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4.4 Rain on a long channel with supercritical flow

PARAMETERS OF THE SOLUTION
Length of the domain: 1000 meters
Width of the domain: 10 meters
Space step along x: 2 meters
Space step along y: 2 meters

Rain intensity for t >= 1500 s: 0.001 m/s
Final time: 3000 s

Initial conditions: h = 0 m and q = 0 m^2/s
Imposed discharge on the left boundary: 2.5 m^2/s
Imposed water height on the left boundary: 0.741514 m
Darcy-Weisbach's friction coefficient: 0.065

MacDonald's solution - supercritical-supercritical with rain with Darcy-Weisbach law

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4.5 Dam break

PARAMETERS OF THE SOLUTION
Length of the domain: 10 meters
Width of the domain: 10 meters
Position of the dam: x=5 meters
Space step along x: 0.01 meter
Space step along y: 1 meter
Time value: 6 seconds

Ritter's solution dam break on a dry soil

4.6 Planar surface in a paraboloid (Thacker's solution)

PARAMETERS OF THE SOLUTION

Length of the domain: 4 meters
Width of the domain: 4 meters
Space step in x: 0.02 meters
Space step in y: 0.02 meters
Number of cells in x: 200
Number of cells in y: 200
Topography: $z(x) = h0(((x - L/2)^2 + (y - l/2)^2)/a^2 - 1)$, with h0=0.1 meters and a=1 meters
Time value: 13.4571 seconds