Explanations concerning fluxes components

FullSWOF team

April 7, 2013

In two dimensions, the shallow water equations write

$$\left\{\begin{array}{l}\partial_t h + \partial_x(hu) + \partial_y(hv) = R - I\\ \partial_t(hu) + \partial_x(gh^2/2 + hu^2) + \partial_y(huv) = gh(S_{0x} - S_{fx})\\ \partial_t(hv) + \partial_x(huv) + \partial_y(gh^2/2 + hv^2) = gh(S_{0y} - S_{fy})\end{array}\right. , \tag{1}$$

with $h[m]$ the water height, $\mathbf{u} = (u, v)$ the velocity vector, $R[m/s]$ the rainfall intensity, $I[m/s]$ the infiltration rate, $\mathbf{S_0} = (S_{0x}, S_{0y}) = (-\partial_x z, -\partial_y z)$ the opposite of the slope and $\mathbf{S_f} = (S_{fx}, S_{fy})$ the friction term. We can write this system under a vector conservative form

$$\partial_t U + \partial_x F(U) + \partial_y G(U) = S(U), \tag{2}$$

with

$$U = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix},\ F(U) = \begin{pmatrix} f_1(h,u,v) \\ f_2(h,u,v) \\ f_3(h,u,v) \end{pmatrix} = \begin{pmatrix} hu \\ gh^2/2 + hu^2 \\ huv \end{pmatrix}, \tag{3}$$

$$G(U) = \begin{pmatrix} g_1(h,u,v) \\ g_2(h,u,v) \\ g_3(h,u,v) \end{pmatrix} = \begin{pmatrix} hv \\ huv \\ gh^2/2 + hv^2 \end{pmatrix} = \begin{pmatrix} f_1(h,v,u) \\ f_3(h,v,u) \\ f_2(h,v,u) \end{pmatrix} \tag{4}$$

and

$$S(U) = \begin{pmatrix} R - I \\ S_{0x} - S_{fx} \\ S_{0y} - S_{fy} \end{pmatrix}. \tag{5}$$

We can notice that, there is no need to use two flux functions in both directions. We just need to permute the last two components and the velocity components.